

**BAREM DE EVALUARE ȘI DE NOTARE****Varianta 15**

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- ♦ Pentru orice soluție corectă, chiar dacă este diferită de cea din barem, se acordă punctajul corespunzător.
- ♦ Nu se acordă fracțiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parțiale, în limitele punctajului indicat în barem.
- ♦ Se acordă 10 puncte din oficiu. Nota finală se calculează prin împărțirea punctajului obținut la 10.

**SUBIECTUL I (30 de puncte)**

|    |                                                                                                                                                                                                   |                      |
|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|
| 1. | $\log_2(5 - \sqrt{3}) + \log_2(5 + \sqrt{3}) - \log_2 11 = \log_2 22 - \log_2 11 =$<br>$= \log_2 2 = 1$                                                                                           | 3p<br>2p             |
| 2. | $S = 2(1 + 2 + 3 + \dots + 2012) - 2012 =$<br>$= 2 \cdot \frac{2012 \cdot 2013}{2} - 2012 =$<br>$= 2012^2$                                                                                        | 2p<br>2p<br>1p       |
| 3. | $2^{x^2+x+0,5} = 2^{\frac{5}{2}} \Leftrightarrow$<br>$x^2 + x - 2 = 0 \Leftrightarrow$<br>$\Leftrightarrow x \in \{-2, 1\}$                                                                       | 1p<br>2p<br>2p       |
| 4. | $x \in \mathbb{N}, 2 \leq x \leq 10$<br>Formula de calcul a combinațiilor<br>$x \geq 6$<br>$\Rightarrow x \in \{6, 7, 8, 9, 10\}$                                                                 | 1p<br>1p<br>2p<br>1p |
| 5. | Formula pentru coordonatele mijlocului unui segment<br>$A(2, 2), B(-2, -2)$ și $C(4, 0)$                                                                                                          | 2p<br>3p             |
| 6. | $\cos^2 \alpha = 1 - \sin^2 \alpha =$<br>$= \left(\frac{5}{13}\right)^2$<br>$\alpha \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \cos \alpha < 0$<br>$\Rightarrow \cos \alpha = -\frac{5}{13}$ | 1p<br>1p<br>1p<br>2p |

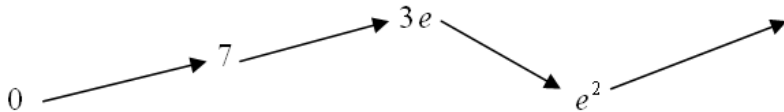
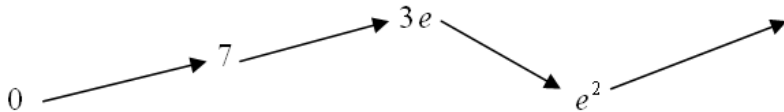
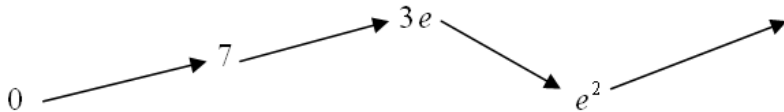
**SUBIECTUL al II-lea (30 de puncte)**

|    |                                                         |    |
|----|---------------------------------------------------------|----|
| 1. | $\det A(x) = x(x^2 + 3)$                                | 3p |
| a) | $A(x)$ inversabilă $\Leftrightarrow x \in \mathbb{R}^*$ | 2p |

|    |                                                                                                                                                                                                                                                                                                                                                             |                                  |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|
| b) | $\det A(1) = 4 \Rightarrow A(1) \text{ inversabilă}$<br>$A^*(1) = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 2 & 0 & 2 \end{pmatrix}$<br>$A^{-1}(1) = \frac{1}{d} A^*(1) = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$                                                    | 1p<br><br>2p<br><br>2p           |
| c) | $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}(1) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} =$<br>$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$                                                                                                                                                                                                  | 3p<br><br>2p                     |
| 2. | $U(\mathbb{Z}_6) = \{\hat{1}, \hat{5}\}$                                                                                                                                                                                                                                                                                                                    | 2p                               |
| a) | $\mathbb{Z}_6 - U(\mathbb{Z}_6) = \{\hat{0}, \hat{2}, \hat{3}, \hat{4}\}$                                                                                                                                                                                                                                                                                   | 1p                               |
|    | $S = \hat{3}$                                                                                                                                                                                                                                                                                                                                               | 2p                               |
| b) | $\det A = \hat{3}x + 4$<br>$\det A = \hat{1} \Leftrightarrow \hat{3}x + 4 = \hat{1} \Leftrightarrow \hat{3}x = \hat{3} \Leftrightarrow x \in \{\hat{1}, \hat{3}, \hat{5}\}$<br>$\det A = \hat{5} \Leftrightarrow \hat{3}x + 4 = \hat{5} \Leftrightarrow \hat{3}x = \hat{1} \Leftrightarrow x \in \Phi$<br>$\Rightarrow x \in \{\hat{1}, \hat{3}, \hat{5}\}$ | 2p<br><br>1p<br><br>1p<br><br>1p |
| c) | $(x, y) \in \{(\hat{1}, \hat{2})\}$                                                                                                                                                                                                                                                                                                                         | 5p                               |

**SUBIECTUL al III-lea (30 de puncte)**

|    |                                                                                                                                                                                                                                                   |                      |
|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|
| 1. |                                                                                                                                                                                                                                                   |                      |
| a) | $l = \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2 + 5x + 7}{e^x}$<br>Se aplică regula lui l'Hospital de două ori și se obține<br>$l = 0 \Rightarrow$<br>$\Rightarrow d : y = 0$ asimptotă orizontală spre $-\infty$ | 1p<br>2p<br>1p<br>1p |
| b) | F derivabilă pe $\mathbb{R}$ și $f'(x) = e^x(x^2 - 3x + 2)$<br>$f'(x) = 0 \Leftrightarrow x \in \{1, 2\}, f(1) = 3e, f(2) = e^2$                                                                                                                  | 1p<br>1p             |

|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                      |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|-----------|-----|---|---|---|---|---|---|---|-----|---|-----|---|-----------|---|---|---|---|---|-----|---|-----|---|---|--|--|--|--|--|-----------|--|-----------|---------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--------|------------------------------------------------------------------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|-----------|----|
|         | <table><tr><td><math>x</math></td><td><math>-\infty</math></td><td colspan="10"><math>0</math></td><td colspan="10"><math>1</math></td><td colspan="10"><math>2</math></td><td><math>+\infty</math></td></tr><tr><td><math>f(x)'</math></td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>0</td><td>-</td><td>-</td><td>-</td><td>-</td><td>0</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td></tr><tr><td><math>f(x)</math></td><td colspan="15"></td><td><math>+\infty</math></td></tr></table> | $x$                  | $-\infty$ | $0$ |   |   |   |   |   |   |   |     |   | $1$ |   |           |   |   |   |   |   |     |   | $2$ |   |   |  |  |  |  |  |           |  | $+\infty$ | $f(x)'$ | + | + | + | + | + | + | + | + | + | + | + | 0 | - | - | - | - | 0 | + | + | + | + | + | + | + | + | + | $f(x)$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $+\infty$ | 2p |
| $x$     | $-\infty$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | $0$                  |           |     |   |   |   |   |   |   |   | $1$ |   |     |   |           |   |   |   |   |   | $2$ |   |     |   |   |  |  |  |  |  | $+\infty$ |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| $f(x)'$ | +                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | +                    | +         | +   | + | + | + | + | + | + | + | 0   | - | -   | - | -         | 0 | + | + | + | + | +   | + | +   | + | + |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| $f(x)$  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                      |           |     |   |   |   |   |   |   |   |     |   |     |   | $+\infty$ |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
|         | $\Rightarrow 1 - \text{maxim local}, 2 - \text{minim local}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 1p                   |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| c)      | $f(0) = 7, f(1) = 3e, f(2) = e^2$<br>$7 \leq e^2$<br>Conform tabelului de la b) $\Rightarrow 7 \leq f(x) \leq 3e, (\forall) x \in [0, 2]$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 1p<br>1p<br>3p       |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| 2.      | $f$ continuă                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 1p                   |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| a)      | $\int_{-1}^0 \sin x dx = -1 + \cos 1$<br>$\int_0^1 \frac{x}{x+2} dx = x \Big _0^1 - 2 \ln(x+2) \Big _0^1 =$<br>$= 1 - 2 \ln \frac{3}{2}$<br>$I = \cos 1 - 2 \ln \frac{3}{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 1p<br>1p<br>1p<br>1p |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| b)      | $I = \int_{-\pi}^0 \sin^2 x dx$<br>$I = \frac{\pi}{2}$<br>$V = \pi \int_{-\pi}^0 f^2(x) dx = \pi \int_{-\pi}^0 \sin^2 x dx = \frac{\pi^2}{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 2p<br>1p<br>2p       |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |
| c)      | $\int_0^x f(t) dt = x - 2 \ln(x+2) + 2 \ln 2$<br>$\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x f(t) dt = 1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2p<br>3p             |           |     |   |   |   |   |   |   |   |     |   |     |   |           |   |   |   |   |   |     |   |     |   |   |  |  |  |  |  |           |  |           |         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |        |                                                                                    |  |  |  |  |  |  |  |  |  |  |  |  |  |  |           |    |