BAREM DE EVALUARE ŞI DE NOTARE

Varianta 24

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SUBIECTUL I (30 de puncte)

1.	$(1+i)^4 = -4$; $(1-i)^4 = -4$; $(1+i)^{2012} - (1-i)^{2012} = (-4)^{503} - (-4)^{503} = 0$	**1p** **1p** **3p**
2.	$\begin{cases} 11x+4 \geq 0 \\ x-2 \geq 0 \end{cases} \Rightarrow x \geq 2$; $x^2 - 15x = 0 \Leftrightarrow x = 0$ şi $x = 15$; Soluţia ecuaţiei: $x = 15$	**2p** **2p** **1p**
3.	$a_6 = a_3 + 3r$; $a_{16} = a_{19} - 3r$; $a_3 + a_{19} = a_6 + a_{16} = 2012$	**1p** **1p** **3p**
4.	$x^2 - 1 = 0 \Leftrightarrow x = \pm 1$; $x + 2 = 0 \Leftrightarrow x = -2$; $x \in [-2,-1] \cup [1,+\infty]$	**2p** **2p** **1p**
5.	Fie M mijlocul segmentului [AB] $\Rightarrow$ M(-1,2); $m_{AB} = -1 \Rightarrow m' = 1$ Ecuaţia mediatoarei lui [AB]: $x - y + 3 = 0$	**1p** **2p** **2p**
6.	$\sin^2 x - \cos^2 x = \cos x \Leftrightarrow 2\cos^2 x + \cos x - 1 = 0$; $\cos x = -1 \Rightarrow x \in \{\pi + 2k\pi, k \in Z\}$; $\cos x = \frac{1}{2} \Rightarrow x \in \{\pm\frac{\pi}{3} + 2k\pi, k \in Z\}$; $x \in [0,2\pi] \Rightarrow x \in \{\pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$	**1p** **1p** **2p** **1p**

x	$-\infty$	-2	-1	1	$+\infty$
$x+2$	$-$	0	$+$	$+$	$+$
x^2-1	$+$	0	$-$	0	$+$
$(x+2)(x^2-1)$	$-$	0	$+$	0	$-$

SUBIECTUL al II-lea (30 de puncte)

1. **a)**	$\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -1, \Rightarrow rangM \geq 2, \forall m \in \mathbb{R}$; $detM = m^2 - 6m + 5$; $rangM = 2 \Leftrightarrow detM = 0 \Leftrightarrow m = 1$ sau $m = 5$	**2p** **2p** **1p**
b)	A,B,C sunt necoliniare $\Leftrightarrow detM \neq 0$; $m^2 - 6m + 5 \neq 0 \Leftrightarrow m \in \mathbb{R} \setminus \{1,5\}$	**3p** **2p**
c)	$A_{ABC} = \frac{1}{2}\lvert det\, M\rvert = \frac{1}{2}\lvert m^2 - 6m + 5\rvert$; $m \in [1,5] \Rightarrow 0 \geq m^2 - 6m + 5 \geq -4$; A_{ABC} maximă $= 2$	**2p** **2p** **1p**
2. **a)**	Observăm că $x * y = \frac{1}{5}[(5x+6)(5y+6) - 6]; \forall x, y \in \mathbb{Z}$;	**1p**

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	$(x*y)*z=\frac{1}{5}\left[(5x+6)(5y+6)(5z+6)-6\right]=x*(y*z),\ \forall x,y,z\in\mathbb{Z};$	**3p**
	$\Leftrightarrow *$ este asociativă	**1p**
b)	Elementul neutru al operaţiei $*$ este $e=-1\in\mathbb{Z}$;	**1p**
	$x*x'=e=\frac{1}{5}[(5x+6)(5x'+6)-6]=-1;$	**1p**
	$\Rightarrow 5x'+6=\frac{1}{5x+6}$	**1p**
	Cum $x'\in\mathbb{Z}\Rightarrow\frac{1}{5x+6}\in\mathbb{Z}\Rightarrow 5x+6\in\{-1,1\};$	**1p**
	$\Rightarrow 5x\in\{-7,-5\}$ .Deci x = -1 este simetrizabil şi $x'=-1$	**1p**
c)	Observăm că $x*x=\frac{1}{5}\left[(5x+6)^2-6\right];\forall x\in\mathbb{Z};$	**1p**
	Inductiv obţinem $\underbrace{x*x*...*x}_{\text{de 2012 ori}}=\frac{1}{5}\left[(5x+6)^{2012}-6\right];$	**2p**
	$\frac{1}{5}\left[(5x+6)^{2012}-6\right]=-1;$	**1p**
	$\Leftrightarrow 5x+6=1\Leftrightarrow x=-1\in\mathbb{Z}$	**1p**

SUBIECTUL al III-lea (30 de puncte)

1.	$f'(x)=(x+2)e^x,\forall x\in\mathbf{R};$	**2p**
a)	$f'(x)=0\Rightarrow x=-2;$	**1p**
	Pe $(-\infty,-2]$ f este strict descrescătoare;	**1p**
	Pe $[-2,+\infty)$ f este strict crescătoare	**1p**
b)	$f''(x)=(x+3)e^x,\forall x\in\mathbf{R};$	**1p**
	$f''(x)=0\Rightarrow x=-3;$	**2p**
	Pe $(-\infty,-3]$ f este concavă;	
	Pe $[-3,+\infty)$ f este convexă	**2p**
c)	$\lim\limits_{x\to-\infty}f(x)=\lim\limits_{x\to-\infty}(x+1)e^x=\lim\limits_{x\to-\infty}\frac{x+1}{e^{-x}}=0;$	**3p**
	y = 0 ; AO spre $-\infty$	**2p**
2.	$F(x)=3x^2+2\ln x+C;\ \forall x\in[1,+\infty);$	**2p**
a)	$F(1)=2012\Rightarrow C=2009;$	**2p**
	$F(x)=3x^2+2\ln x+2009$	**1p**
b)	$V=\pi\int_1^2 f^2(x)dx=\pi(12x^3+24x-4x^{-1})\Big\vert_1^2;$	**3p**
	$V=110\pi$	**2p**
c)	$\lim\limits_{x\to\infty}\frac{f(x)}{x}=6=m;$	**2p**
	$\lim\limits_{x\to\infty}\left[f(x)-mx\right]=\lim\limits_{x\to\infty}\frac{2}{x}=0=n;$	**1p**
	y = 6x este asimptota oblică către $+\infty$ a graficului funcţiei f.	**2p**

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