

BAREM DE EVALUARE ȘI DE NOTARE**Varianta 25**

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SUBIECTUL I (30 de puncte)

1.	$(1+i)^4 = -4$; $(1+i)^4 = -4$; $(1+i)^{2012} - (1-i)^{2012} = (-4)^{503} - (-4)^{503} = 0$	1p 1p 3p
2.	Notăm $3^x = y \Rightarrow 3y^2 - 10y + 3 = 0 \Rightarrow y_1 = 3; y_2 = 1/3$; $3^x = 3 \Rightarrow x = 1$; $3^x = 1/3 \Rightarrow x = -1$	3p 1p 1p
3.	$a_6 = a_3 + 3r$; $a_{16} = a_{19} - 3r$; $a_3 + a_{19} = a_6 + a_{16} = 2012$	1p 1p 3p
4.	$C_{n+1}^1 + C_{n+1}^2 = 36 \Rightarrow (n+1)(n+2) = 72$; $n+1 = 8 \Rightarrow n = 7$	3p 2p
5.	Fie M mijlocul segmentului $[AB] \Rightarrow M(-1,1)$; $m_{AB} = -3/4 \Rightarrow m' = 4/3$ Ecuația mediatoarei lui $[AB]$: $4x - 3y + 7 = 0$	2p 1p 2p
6.	$\sin^2 x - \cos^2 x = \cos x \Leftrightarrow 2\cos^2 x + \cos x - 1 = 0$; $\cos x = -1 \Rightarrow x \in \{\pi + 2k\pi, k \in \mathbb{Z}\}$; $\cos x = \frac{1}{2} \Rightarrow x \in \{\pm \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\}$; $x \in [0, 2\pi] \Rightarrow x \in \{\pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$	1p 1p 2p 1p

SUBIECTUL al II-lea (30 de puncte)

1. a)	$H^*(t) = \begin{pmatrix} t & -t \ln t & 0 \\ 0 & t & 0 \\ 0 & 0 & 1 \end{pmatrix}; \forall t > 0$; $\det H^*(t) = t^2$	3p 2p
b)	$H(x) \cdot H(y) = \begin{pmatrix} 1 & \ln x + \ln y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & xy \end{pmatrix}; \forall x, y \in (0, +\infty)$; Deci $H(x) \cdot H(y) = H(xy); \forall x, y \in (0, +\infty)$	3p 2p
c)	$H(1)+H(2)+H(3)+\dots+H(10) = \begin{pmatrix} 10 & \ln(1 \cdot 2 \cdot \dots \cdot 10) & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 55 \end{pmatrix}$; $\det[H(1)+H(2)+H(3)+\dots+H(10)] = 5500$	3p 2p
2. a)	$\left. \begin{matrix} x > 2 \Rightarrow x - 2 > 0 \\ y > 2 \Rightarrow y - 2 > 0 \end{matrix} \right\} \Rightarrow xy - 2x - 2y + 4 > 0$	3p

	$x * y = xy - 2x - 2y + 6 \in G; \forall x, y \in G;$ G este parte stabilă față de operația *	1p 1p
b)	Observăm că operația * este comutativă; Elementul neutru: $e = 3;$ $x * x' = 3 \Rightarrow x'(x - 2) = 2x - 3, \forall x \in G;$ $x' = 2 + \frac{1}{x - 2} > 0, \forall x \in G$	1p 1p 1p 2p
c)	$x * y * z = (x - 2)(y - 2)(z - 2) + 2, \forall x, y, z \in G$ $\frac{1}{2} * \frac{2}{3} * \dots * \frac{8}{9} = \frac{-3}{2} \cdot \frac{-4}{3} \cdot \dots \cdot \frac{-10}{9} + 2 = 7$	2p 3p

SUBIECTUL al III-lea (30 de puncte)

1.	$f'(x) = 2012x^{2011} + 2012, \forall x \in \mathbb{R};$	2p
a)	$f(1) = 0;$ $f'(0) = 2012;$ $f(1) + f'(0) = 2012$	1p 1p 1p
b)	$y - f(1) = f'(1)(x - 1);$ $y = 4024(x - 1)$	3p 2p
c)	$f''(x) = 2012 \cdot 2011x^{2010}, \forall x \in \mathbb{R};$ $f''(x) > 0, \forall x \in \mathbb{R} \Leftrightarrow f$ este convexă	3p 2p
2.	$f(x) = x^3 + 3x, \forall x \in \mathbf{R} \quad (1)$	1p
a)	$I = \int_0^1 f(x)dx = \int_0^1 (x^3 + 3x)dx = \frac{x^4}{4} + \frac{3x^2}{2} \Big _0^1;$ $I = \frac{7}{4}$	3p 1p
b)	$f^5(-x) = [(-x)^3 + 3(-x)]^5 = -f^5(x), \forall x \in \mathbf{R};$ f^5 este funcție impară $\Rightarrow \int_{-1}^1 f^5(x)dx = 0$	3p 2p
c)	$\int_0^x f(t-1)dt = \frac{(t-1)^4}{4} + \frac{3(t-1)^2}{2} \Big _0^x = \frac{(x-1)^4 + 6(x-1)^2 - 7}{4};$ $\lim_{x \rightarrow \infty} \frac{\int_0^x f(t-1)dt}{x^4} = \lim_{x \rightarrow \infty} \frac{(x-1)^4 + 6(x-1)^2 - 7}{4x^4} = \frac{1}{4}$	3p 1p