BAREM DE EVALUARE ŞI DE NOTARE

Varianta 64

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- **Pentru orice soluţie corectă, chiar dacă este diferită de cea din barem, se acordă punctajul corespunzător.**
- **Nu se acordă fracţiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parţiale, în limitele punctajului indicat în barem.**
- **Se acordă 10 puncte din oficiu. Nota finală se calculează prin împărţirea punctajului obţinut la10.**

SUBIECTUL I (30 de puncte)

1.	$\begin{cases} a_1=2 \\ a_5=14 \end{cases} \Leftrightarrow \begin{cases} a_1=2 \\ a_1+4r=14 \end{cases} \Leftrightarrow \begin{cases} a_1=2 \\ r=3 \end{cases}$ $\Rightarrow a_{100}=a_1+99r=299$ $\Rightarrow S_{100}=\frac{(a_1+a_{100})\cdot 100}{2}=15050$	3p 1p 1p
2.	$\begin{cases} x_1+x_2=m-1 \\ x_1x_2=m-1 \end{cases}$ Rel.devine: $\sqrt{m-1}+\sqrt{9-(m-1)}=3 \Leftrightarrow \sqrt{m-1}+\sqrt{10-m}=3(*)$ Cond.: $\begin{cases} m-1\geq 0 \\ 10-m\geq 0 \end{cases} \Leftrightarrow \begin{cases} m\geq 1 \\ m\leq 10 \end{cases} \Rightarrow m\in[1;10]$ Ridicand la pătrat în ambii membri,ecuatia (*)devine: $m-1+2\sqrt{m-1}\cdot\sqrt{10-m}+10-m=9 \Leftrightarrow \sqrt{m-1}\cdot\sqrt{10-m}=0$ $\sqrt{m-1}=0\Rightarrow m=1\in[1;10]$ sau $\sqrt{10-m}=0\Rightarrow m=10\in[1;10]$	1p 1p 1p 2p
3.	Cond.ca logaritmii sa fie corect definiti:	1p

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	$\begin{cases} 2x-5>0 \\ x^2-8>0 \\ \log_2(x^2-8)\neq 0 \end{cases} \Leftrightarrow \begin{cases} x\in(\frac{5}{2};+\infty) \\ x\in(-\infty;-2\sqrt{2})\cup(2\sqrt{2};+\infty)\Rightarrow D=(2\sqrt{2};+\infty)-\{10\} \\ x\neq\pm 10 \end{cases}$	2p
	Ec.devine $2\log_2(2x-5)=\log_2(x^2-8)\Leftrightarrow\log_2(2x-5)^2=\log_2(x^2-8)\Leftrightarrow(2x-5)^2=x^2-8$	2p
	$\Rightarrow 3x^2-20x+33=0\Rightarrow x_1=3\in D$ si $x_2=\frac{11}{3}\in D$	
4.	$(1+i)^4=(1+i)^2\cdot(1+i)^2=(1+2i+i^2)(1+2i+i^2)=$	3p
	$=2i\cdot 2i=4i^2=-4$	2p
	$\text{Re}((1+i)^4)=-4$	
5.	$C_{12}^3=C_{12}^9\left(combinari\ complementare\right)$	3p
	Finalizare: $C_{12}^3-C_{12}^9=0$	2p
6.	Notăm cu *h* înălţimea dusă din A pe latura BC $\Rightarrow m_h\cdot m_{BC}=-1$	1p
	$m_{BC}=\frac{y_B-y_C}{x_B-x_C}=\frac{3+2}{-4-6}=\frac{5}{-10}=-\frac{1}{2}\Rightarrow m_h\cdot\left(-\frac{1}{2}\right)=-1\Rightarrow m_h=2$	2p
	$y-y_0=m(x-x_0)\Rightarrow$ y-6=2(x-2)	2p

SUBIECTUL al II-lea (30 de puncte)

1. a)	Fie $A=\begin{pmatrix} \hat{a} & \hat{b} \\ \hat{2}\cdot\hat{b} & \hat{a} \end{pmatrix}\in M, \hat{a},\hat{b}\in Z_5$; luam $\hat{a}=\hat{1}$ si $\hat{b}=\hat{0}\Rightarrow A=\begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & \hat{1} \end{pmatrix}=I_2$	2p

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	Daca luam $\hat{a}=\hat{b}=\hat{0}\Rightarrow A=\begin{pmatrix}\hat{0} & \hat{0}\\ \hat{0} & \hat{0}\end{pmatrix}=O_2$ $\Rightarrow I_2, O_2 \in M$.	2p 1p
b)	$Fie\ A=\begin{pmatrix}\hat{a} & \hat{b}\\ \hat{2}\cdot\hat{b} & \hat{a}\end{pmatrix}\in M, \hat{a},\hat{b}\in Z_5$ si $B=\begin{pmatrix}\hat{c} & \hat{d}\\ \hat{2}\cdot\hat{d} & \hat{c}\end{pmatrix}\in M; \hat{c},\hat{d}\in Z_5$ $A+B=\begin{pmatrix}\hat{a}+\hat{c} & \hat{b}+\hat{d}\\ \hat{2}\cdot\hat{b}+\hat{2}\hat{d} & \hat{a}+\hat{c}\end{pmatrix}=\begin{pmatrix}\hat{a}+\hat{c} & \hat{b}+\hat{d}\\ \hat{2}\cdot\left(\hat{b}+\hat{d}\right) & \hat{a}+\hat{c}\end{pmatrix}=\begin{pmatrix}\hat{m} & \hat{n}\\ \hat{2}\cdot\hat{n} & \hat{m}\end{pmatrix}\in M$, unde $\hat{m}=\hat{a}+\hat{c}\in\mathbb{Z}_5$ si $\hat{n}=\hat{b}+\hat{d}\in\mathbb{Z}_5$ $A\cdot B=\begin{pmatrix}\hat{a}\hat{c}+\hat{2}\hat{b}\hat{d} & \hat{a}\hat{d}+\hat{b}\hat{c}\\ \hat{2}\hat{b}\hat{c}+\hat{2}\hat{a}\hat{d} & \hat{a}\hat{c}+\hat{2}\hat{b}\hat{d}\end{pmatrix}=\begin{pmatrix}\hat{a}\hat{c}+\hat{2}\hat{b}\hat{d} & \hat{a}\hat{d}+\hat{b}\hat{c}\\ \hat{2}\cdot\left(\hat{a}\hat{d}+\hat{b}\hat{c}\right) & \hat{a}\hat{c}+\hat{2}\hat{b}\hat{d}\end{pmatrix}=\begin{pmatrix}\hat{p} & \hat{q}\\ \hat{2}\cdot\hat{q} & \hat{p}\end{pmatrix}\in M$, unde $\hat{p}=\hat{a}\hat{c}+\hat{2}\hat{b}\hat{d}\in\mathbb{Z}_5$ si $\hat{q}=\hat{a}\hat{d}+\hat{b}\hat{c}\in\mathbb{Z}_5$	2p 3p
c)	A ∈ M este inversabilă $\Leftrightarrow \det(A)\neq\hat{0}$ $\hat{a}^2-\hat{2}\hat{b}^2\neq\hat{0}\Leftrightarrow\hat{a}^2\neq\hat{2}\hat{b}^2$ Pentru $\hat{a}=\hat{0}\Rightarrow\hat{2}\hat{b}^2\neq\hat{0}\Rightarrow\hat{b}\in\left\{\hat{1},\hat{2},\hat{3},\hat{4}\right\}\Rightarrow$ se construiesc 4 matrice inversabile Pentru $\hat{a}=\hat{1}\Rightarrow\hat{2}\hat{b}^2\neq\hat{1}\Rightarrow\hat{b}\in\left\{\hat{0},\hat{1},\hat{2},\hat{3},\hat{4}\right\}\Rightarrow$ se construiesc 5 matrice inversabile Pentru $\hat{a}=\hat{2}\Rightarrow\hat{2}\hat{b}^2\neq\hat{2}^2\Rightarrow\hat{2}\hat{b}^2\neq\hat{4}\Rightarrow\hat{b}\in\left\{\hat{0},\hat{1},\hat{2},\hat{3},\hat{4}\right\}\Rightarrow$ se construiesc 5 matrice inversabile	1p 2p

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	Pentru $\hat{a}=\hat{3}\Rightarrow \hat{2}\hat{b}^2\neq\hat{3}^2\Rightarrow \hat{2}\hat{b}^2\neq\hat{4}\Rightarrow \hat{b}\in\{\hat{0},\hat{1},\hat{2},\hat{3},\hat{4}\}\Rightarrow$ se construiesc 5 matrice inversabile Pentru $\hat{a}=\hat{4}\Rightarrow \hat{2}\hat{b}^2\neq\hat{4}^2\Rightarrow \hat{2}\hat{b}^2\neq\hat{1}\Rightarrow \hat{b}\in\{\hat{0},\hat{1},\hat{2},\hat{3},\hat{4}\}\Rightarrow$ se construiesc 5 matrice inversabile Deci mulţimea U(M) = {A $\in$ M | există $A^{-1}\in M$ }are 24 elemente	2p
2. a)	$\left(X^2-\frac{1}{2}\right)^2+\left(X+\frac{1}{2}\right)^2+\frac{1}{2}=X^4-X^2+\frac{1}{4}+X^2+X+\frac{1}{4}+\frac{1}{2}=$ $=X^4+X+1=f$	4p 1p
b)	x_1 rădăcină a polin.$f\Rightarrow f(x_1)=0\Rightarrow x_1^4+x_1+1=0(1)$ x_2 rădăcină a polin.$f\Rightarrow f(x_2)=0\Rightarrow x_2^4+x_2+1=0(2)$ x_3 rădăcină a polin.$f\Rightarrow f(x_3)=0\Rightarrow x_3^4+x_3+1=0(3)$ x_4 rădăcină a polin.$f\Rightarrow f(x_4)=0\Rightarrow x_4^4+x_4+1=0(4)$ Însumând cele 4 relaţii$\Rightarrow \left(x_1^4+x_2^4+x_3^4+x_4^4\right)+\left(x_1+x_2+x_3+x_4\right)+4=0$,dar cf.rel.Viete ştim că $x_1+x_2+x_3+x_4=0\Rightarrow x_1^4+x_2^4+x_3^4+x_4^4=-4$	3p 1p 1p
c)	Conform pct.*a)* $f=\underbrace{\left(x^2-\frac{1}{2}\right)^2}_{\geq 0}+\underbrace{\left(x+\frac{1}{2}\right)^2}_{\geq 0}+\frac{1}{2}>0,\forall x\in\mathbb{R}$ Polinomul f nu admite rădăcini reale.	3p 2p

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SUBIECTUL al III-lea (30 de puncte)

1. a) $\lim\limits_{x\to\pm\infty} f(x)=+\infty$;f este continuă pe R,derivabilă pe R şi $f'(x)=e^x-1$ — 2p

x	$-\infty$	0	$+\infty$
$f'(x)$	- - - - - - - - - - - - -	0	+ + + + + + + + + + + + + + + + + + + +
f(x)	$+\infty$	↘ ↘ ↘ ↘ ↘	0 ↗ ↗ ↗ ↗ ↗ $+\infty$

2p

finalizare — 1p

b) Din tabelul de la punctul a) se observă că f are un minim global egal cu 0 atins pentru x=0 — 3p

$\Rightarrow f(x)\geq 0,\forall x\in\mathbb{R}$ — 2p

c) Din b) $\Rightarrow f(x)\geq 0,\forall x\in\mathbb{R}\Rightarrow e^x\geq x+1,\forall x\in\mathbb{R}\ (1)$; — 2p

Cf.rel.(1) $\Rightarrow e^{x^2}\geq x^2+1,\forall x\in[0;1]\Rightarrow \dfrac{1}{x^2+1}\geq\dfrac{1}{e^{x^2}}=e^{-x^2}$ (2)

Deoarece $x\in[0;1]\Leftrightarrow 0\leq x\leq 1\Rightarrow x-1\leq 0\,/\cdot(x+1)>0\Rightarrow$ — 2p

$x^2-1\leq 0\Leftrightarrow x^2\leq 1/\cdot(-1)\Rightarrow -x^2\geq -1$

$\Rightarrow e^{-1}\leq e^{-x^2}\ (3)$;

Din (2)si (3) $\Rightarrow e^{-1}\leq e^{-x^2}\leq\dfrac{1}{x^2+1}$ (prin integrare) $\Rightarrow \int_0^1 e^{-1}dx\leq\int_0^1 e^{-x^2}dx\leq\int_0^1\dfrac{1}{x^2+1}dx\Rightarrow$ — 1p

$\dfrac{1}{e}\cdot x\Big|_0^1\leq\int_0^1 e^{-x^2}dx\leq arctgx\Big|_0^1\Rightarrow\dfrac{1}{e}\leq\int_0^1 e^{-x^2}dx\leq\dfrac{\pi}{4}$

2. a) Funcţia *f* are minim(a=1>0) şi $V_f\left(\dfrac{a}{2};-\dfrac{a}{4}\right)$ — 2p

Funcţia *g* are maxim (a= -1<0) şi $V_g\left(\dfrac{3a}{2};\dfrac{9a^2}{4}\right)$ — 3p

b) Studiem semnul diferenţei $f(x)-g(x)=2x^2-4ax$

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$\Rightarrow$

$f(x)-g(x)=0 \Leftrightarrow 2x^2-4ax=0 \Rightarrow \begin{cases} x_1=0 \Rightarrow f(0)=g(0)=0 \Rightarrow O(0,0) \\ x_2=2a \Rightarrow f(2a)=g(2a)=2a^2 \Rightarrow P(2a,2a^2) \end{cases}$ 2p

x	0 2a
$f(x)-g(x)$	++++++++++++++++++0 - - - - - - - - - -0++++++++++++++

2p

$\Rightarrow f(x)-g(x)\le 0, \forall x\in[0,2a] \Rightarrow g(x)\ge f(x), \forall x\in[0,2a]$

Calculăm $S=\int_0^{2a}\left(g(x)-f(x)\right)dx=\int_0^{2a}\left(-2x^2+4ax\right)dx=\left(-2\frac{x^3}{3}+4a\frac{x^2}{2}\right)\Big|_0^{2a}=\frac{8a^3}{3}$ 1p

c) Ecuatia dreptei OP: $\begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 2a & 2a^2 & 1 \end{vmatrix}=0 \Rightarrow y-ax=0 \Rightarrow y=ax \overset{not.}{=} h(x)$ 1p

Notăm cu S_1 suprafaţa cuprinsă între curba lui g şi dreapta OP

Notăm cu S_2 suprafaţa cuprinsă între curba lui f şi dreapta OP 2p

$\Rightarrow S_1=\int_0^{2a}\left(g(x)-h(x)\right)dx=\int_0^{2a}\left(2ax-x^2\right)dx=\left(2a\cdot\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_0^{2a}=\frac{4a^3}{3}(1)$

$S_2=\int_0^{2a}\left(h(x)-f(x)\right)dx=\int_0^{2a}\left(2ax-x^2\right)dx=\frac{4a^3}{3}(2)$ 2p

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