BAREM DE EVALUARE ŞI DE NOTARE

Varianta 10

Prof: Băşcău Cornelia

- Pentru orice soluţie corectă, chiar dacă este diferită de cea din barem, se acordă punctajul corespunzător.
- Nu se acordă fracţiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parţiale, în limitele punctajului indicat în barem.
- Se acordă 10 puncte din oficiu. Nota finală se calculează prin împărţirea punctajului obţinut la 10.

SUBIECTUL I (30 de puncte)

1.	$\lg 10^6 = 6, \sqrt{10^6} = 10^3, \sqrt[3]{10^6} = 100$ $\lg 10^6 + \sqrt{10^6} + \sqrt[3]{10^6} = 1106$	3p 2p
2.	$x^2 - 3x - 10 = 0 \Rightarrow x \in \{5, -2\}$ $Gr_f \cap Ox = \{A(-2,0), (5,0)\}$	2p 3p
3.	$C.E. x > 0, \log_{27}^2 x^3 = \left(3\log_{27} x\right)^2$ $not \log_{27} x = t \Rightarrow 9t^2 + 9t - 4 = 0 \Rightarrow t \in \left\{\frac{1}{3}, -\frac{4}{3}\right\}$ $\log_{27} x = \frac{1}{3} \Rightarrow x = \sqrt[3]{27} \Rightarrow x = 3 \in \mathbb{N}$ $\log_{27} x = -\frac{4}{3} \Rightarrow x = \sqrt[3]{27^{-4}} \Rightarrow x = 3^{-4} \notin \mathbb{N}$	1p 2p 1p 1p
4.	$T_{k+1} = C_9^k (3x)^{9-k} \left(\frac{1}{\sqrt{x}}\right)^k \Rightarrow x^{9-k} \left(\frac{1}{\sqrt{x}}\right)^k = x^0$ $x^{9-k-\frac{1}{2}k} = x^0$ $k = 6 \Rightarrow T_{6+1} = C_9^6 3^3 = 2268$	2p 2p 1p
5.	$A(2,0), B(4,2), C(6,-4)$ $x_G = \frac{x_A + x_B + x_C}{3} \Rightarrow x_G = 4$ $y_G = \frac{y_A + y_B + y_C}{3} \Rightarrow y_G = \frac{-2}{3}$	1p 2p 2p

6.	$6 \in \left(\frac{3\pi}{2}, 2\pi\right) \to \sin 6 < 0$ $7 \in \left(0, \frac{\pi}{2},\right) \to \sin 7 > 0$ $\sin 6 < \sin 7$	2p 1p 2p

SUBIECTUL al II-lea (30 de puncte)

1. a)	$f(-1) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, f(1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $f(-1) + f(1) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2$	2p 3p
b)	$f(2x) = \begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix}$ $\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $x = \frac{1}{2}$	2p 2p 1p
c)	$f(x) \cdot f(x) = \begin{pmatrix} x^2 & 0 \\ 0 & x^2 \end{pmatrix}$ $(f(x))^n = \begin{pmatrix} x^n & 0 \\ 0 & x^n \end{pmatrix}, \forall n \in \mathbb{N}^*$ $f(2) + \ldots + (f(2))^{2014} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} + \ldots + \begin{pmatrix} 2^{2014} & 0 \\ 0 & 2^{2014} \end{pmatrix} =$ $\begin{pmatrix} 2 + .. + 2^{2014} & 0 \\ 0 & 2 + .. + 2^{2014} \end{pmatrix} =$ $\begin{pmatrix} 2^{2015} - 2 & 0 \\ 0 & 2^{2015} - 2 \end{pmatrix}$	1p 1p 1p 1p 1p
2. a)	elemente inversabile $\hat{1}, \hat{2}, \hat{4}, \hat{5}, \hat{7}, \hat{8}$ $\hat{1} \cdot \hat{2} \cdot \hat{4} \cdot \hat{5} \cdot \hat{7} \cdot \hat{8} = \hat{8}$	2p 3p
b)	$\hat{1} + \hat{2} + \ldots + \hat{9} = \hat{0}$ $\hat{1} + \hat{2} + \ldots + 2014 = \hat{1} + \hat{2} + \ldots + \hat{7} = \hat{1}$	2p 3p

c)	$\begin{cases}\hat{3}x+\hat{2}y=\hat{0}\\ \hat{4}x+\hat{5}y=\hat{1}\end{cases}\Big\vert\,(\ )\cdot\hat{2}\Rightarrow\begin{cases}\hat{6}x+\hat{4}y=\hat{0}\\ \hat{4}x+\hat{5}y=\hat{1}\end{cases}\Rightarrow$ $10x+\hat{9}y=\hat{1}\Rightarrow x=\hat{1}$ $y=\hat{3}, S=\left\{(\hat{1},\hat{3})\right\}$	2p 2p 1p

SUBIECTUL al III-lea (30 de puncte)

1. a)	$(x^{2014})'=2014x^{2013}$ $(2014^x)'=2014^x\ln 2014$ $f'(x)=2014x^{2013}+2014^x\ln 2014$	2p 2p 1p
b)	$y-y_0=f'(x_0)(x-x_0)$ $x_0=0, y_0=1-\ln 2014$ $f'(1)=\ln 2014$ $\ln 2014x-y+1-\ln 2014=0$	1p 1p 1p 2p
c)	$f''(x)=2014\cdot 2013x^{2012}+2014^x\ln^2 2014$ $x^{2012}\geq 0, 2014^x>0$ $f''(x)>0\Rightarrow fconv.pe\mathbb{R}$	2p 1p 2p
2. a)	$\int_2^4 f(x)dx=\int_2^4\frac{1}{x}dx+\int_2^4\frac{1}{x+2}dx=$ $=\left(\ln\lvert x\rvert+\ln\lvert x+2\rvert\right)\Big\vert_2^4=$ $=\ln 4-\ln 2+\ln 6-\ln 4=\ln\frac{6}{2}=\ln 3$	2p 2p 1p
b)	$Fprim.f\Rightarrow F'(x)=f(x),\forall x\in[1,\infty)$ $F''(x)=f'(x)=-\frac{1}{x^2}-\frac{1}{(x+2)^2}<0$ $F''(x)<0\Rightarrow Fconc.pe[1,\infty)$	2p 2p 1p

c)	$V=\pi\int_1^2\left(\frac{1}{x}+\frac{1}{x+2}\right)^2 dx=$	2p
	$\pi\left(\int_1^2 x^{-2}dx+\int_1^2 (x+2)^{-2}dx+2\int_1^2 \frac{1}{x(x+2)}dx\right)=$	
	$\pi\left(\frac{x^{-1}}{-1}\Big\|_1^2+\frac{(x+2)^{-1}}{-1}\Big\|_1^2+2\int_1^2 \frac{1}{(x+1)^2-1}(x+1)'dx\right)=$	2p
	$\pi\left(1-2^{-1}+3^{-1}-4^{-1}+2\frac{1}{2}\ln\left\|\frac{(x+1)-1}{(x+1)+1}\right\|_1^2\right)=\pi\left(\frac{7}{12}+\ln\frac{3}{2}\right)$	1p