

BAREM DE EVALUARE ȘI DE NOTARE

Varianta 57

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- ◆ Pentru orice soluție corectă, chiar dacă este diferită de cea din barem, se acordă punctajul corespunzător.
- ◆ Nu se acordă fracțiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parțiale, în limitele punctajului indicat în barem.
- ◆ Se acordă 10 puncte din oficiu. Nota finală se calculează prin împărțirea punctajului obținut la 10.

SUBIECTUL I (30 de puncte)

1.	$z + \frac{1}{z} = i, \quad z^4 + \frac{1}{z^4} = \left(z^2 + \frac{1}{z^2} \right)^2 - 2 = \left[\left(z + \frac{1}{z} \right)^2 - 2 \right] - 2 =$ $= (-3)^2 - 2 = 7.$	3p 2p
2.	$(f \circ g)(x) = g^2(x) - 6g(x) + 8$ $g^2(x) - 6g(x) + 8 = 0 \Rightarrow g(x) = 2, g(x) = 4$ $g(x) = 2 \Rightarrow 2x - 6 = 2 \Rightarrow x = 4, \quad g(x) = 4 \Rightarrow 2x - 6 = 4 \Rightarrow x = 5$ $x \in \{4, 5\}.$	1p 2p 1p 1p
3.	$x > 1, \quad \log_{x+1}(x-1) = \frac{1}{\log_{x-1}(x+1)}, \text{ ecuația se poate scrie așa}$ $\log_{x+1}(x-1) + \frac{1}{\log_{x+1}(x-1)} = 2, \text{ dar } \log_{x+1}(x-1) + \frac{1}{\log_{x+1}(x-1)} \geq 2 \Rightarrow$ $\Rightarrow \log_{x+1}(x-1) = \frac{1}{\log_{x+1}(x-1)} = 1 \Rightarrow x-1 = x+1, \text{ ecuația nu are rezolvare în } R.$	1p 2p 2p
4.	<p>Se cunoaște identitatea $C_n^0 + C_n^1 + \dots + C_n^n = 2^n$,</p> $2048 = 2^{11} \Rightarrow n = 11.$	2p 3p
5.	$\overrightarrow{AB}(-1, 2), \quad \overrightarrow{BC}(1, 2)$	2p

	$\overrightarrow{AB} \cdot \overrightarrow{BC} = -1 \cdot 1 + 2 \cdot 2 = 3.$	3p
6.	$2(\sin a + \sin b)(\sin a - \sin b) = 2(\sin^2 a - \sin^2 b) =$ $= 2\left(\frac{1 - \cos 2a}{2} - \frac{1 - \cos 2b}{2}\right) = \cos 2b - \cos 2a.$	2p 3p

SUBIECTUL al II-lea (30 de puncte)

1. a)	<p>După relațiile lui Viète $x_1 + x_2 + x_3 = 0$,</p> $D = \begin{vmatrix} x_1 + x_2 + x_3 & x_2 & x_3 \\ x_1 + x_2 + x_3 & x_1 & x_2 \\ x_1 + x_2 + x_3 & x_3 & x_1 \end{vmatrix} = \begin{vmatrix} 0 & x_2 & x_3 \\ 0 & x_1 & x_2 \\ 0 & x_3 & x_1 \end{vmatrix} = 0.$	2p 3p
b)	<p>Cum $x_1 + x_2 + x_3 = 0$,</p> $x_1^3 + x_2^3 + x_3^3 = 3x_1x_2x_3 = -15.$	2p 3p
c)	$\begin{vmatrix} x & 2-x & x-3 \\ x-3 & x & 2-x \\ 2-x & x-3 & x \end{vmatrix} = \begin{vmatrix} x+2-x+x-3 & 2-x & x-3 \\ x-3+x+2-x & x & 2-x \\ \underline{2-x+x-3+x} & x-3 & x \end{vmatrix} = (x-1) \begin{vmatrix} 1 & 2-x & x-3 \\ 1 & x & 2-x \\ 1 & x-3 & x \end{vmatrix} \begin{matrix} l_2-l_1 \\ l_3-l_1 \end{matrix}$ $(x-1) \begin{vmatrix} 1 & 2-x & x-3 \\ 0 & 2x-2 & 5-2x \\ 0 & 2x-5 & 3 \end{vmatrix} = (x-1)(6x-6+4x^2-20x+25) = (x-1)(4x^2-14x+19)$ $\begin{vmatrix} x & 2-x & x-3 \\ x-3 & x & 2-x \\ 2-x & x-3 & x \end{vmatrix} = 0 \Rightarrow (x-1) \left(\frac{4x^2-14x+19}{\Delta < 0} \right) = 0 \Rightarrow x = 1.$	1p 2p 2p
2. a)	<p>$x * y = (x-4)(y-4) + 4$,</p> <p>$x * e = x \Rightarrow (x-4)(e-4) + 4 = x \Rightarrow (x-4)(e-4) = x-4 \Rightarrow e-4 = 1, e = 5$</p> <p>$x * x' = e \Rightarrow (x-4)(x'-4) + 4 = 5 \Rightarrow x' = \frac{1}{x-4} + 4 = \frac{4x-15}{x-4}.$</p>	1p 2p 2p
b)	<p>$x * b = b * x = b \Rightarrow (x-4)(b-4) + 4 = b \Rightarrow (x-4)(b-4) = b-4 \Rightarrow (b-4)(x-5) = 0$, de unde</p>	3p

	$b = 4.$	2p
c)	Din b). se știe că $4 * x = x * 4 = 4$ $x^4 * \underbrace{4^4}_{=4} * 4 + 4^x = 5 \Leftrightarrow \underbrace{x^4}_{=4} * 4 + 4^x = 5 \Leftrightarrow 4^x + 4 = 5$, de unde $4^x = 1$ $x = 0.$	2p 2p 1p

SUBIECTUL al III-lea (30 de puncte)

1. a)	$a_n = \frac{n^3 + 3n^2 + 3n + 1 - n^3}{[n(n+1)]^3} = \frac{(n+1)^3}{n^3(n+1)^3} - \frac{n^3}{n^3(n+1)^3} = \frac{1}{n^3} - \frac{1}{(n+1)^3}$ $b_n = a_n + \frac{1}{(n+1)^3} = \frac{1}{n^3} - \frac{1}{(n+1)^3} + \frac{1}{(n+1)^3} = \frac{1}{n^3}.$ $\frac{b_{n+1}}{b_n} = \frac{\frac{1}{(n+1)^3}}{\frac{1}{n^3}} = \left(\frac{n}{n+1}\right)^3 < 1 \Rightarrow b_n \text{ este un șir descrescător.}$	2p 1p 2p
b)	$\lim_{n \rightarrow \infty} a_n \cdot n^4 = \lim_{n \rightarrow \infty} \frac{n^4(3n^2 + 3n + 1)}{[n(n+1)]^3} =$ $= \lim_{n \rightarrow \infty} \frac{n^6 \left(3 + \frac{3}{n} + \frac{1}{n^2}\right)}{n^6 \left(1 + \frac{1}{n}\right)} = 3.$	2p 3p
c)	$\lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{n^3} - \frac{1}{(n+1)^3} \right) =$ $= \lim_{n \rightarrow \infty} \left(\underbrace{\frac{1}{1^3} - \frac{1}{2^3}}_{=0} + \underbrace{\frac{1}{2^3} - \frac{1}{3^3}}_{=0} + \underbrace{\frac{1}{3^3} - \frac{1}{4^3}}_{=0} + \dots + \underbrace{\frac{1}{n^3} - \frac{1}{(n+1)^3}}_{=0} \right) =$ $= \lim_{n \rightarrow \infty} \left(1 - \frac{1}{(n+1)^3} \right) = 1.$	1p 2p 2p

2. a)	$I_1 = \int \frac{1+tg^2x}{1-tgx} dx = \int \frac{1}{1-tgx} \cdot \frac{1}{\cos^2 x} dx \stackrel{1-tgx=u}{=} -\int \frac{du}{u} =$ $= -\ln u + C = -\ln tgx-1 + C.$	3p 2p
b)	$I_2 = \int \frac{1+tg^2x}{1-tg^2x} dx = -\int \frac{1}{tg^2x-1} \cdot \frac{1}{\cos^2 x} dx \stackrel{tgx=u}{=} -\int \frac{1}{u^2-1} du =$ $= -\frac{1}{2} \ln \left \frac{u-1}{u+1} \right + C = -\frac{1}{2} \ln \left \frac{tgx-1}{tgx+1} \right + C.$	3p 2p
c)	$\int_0^{\frac{\pi}{6}} f_4(x) dx = \int_0^{\frac{\pi}{6}} \frac{1+tg^2x}{1-tg^4x} dx$ $\frac{1}{1-tg^4x} = -\frac{1}{(tg^2x-1)(tg^2x+1)} = \frac{1}{2} \left(\frac{1}{tg^2x+1} - \frac{1}{tg^2x-1} \right).$ $\int_0^{\frac{\pi}{6}} f_4(x) dx = \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{1+tg^2x}{tg^2x+1} dx + \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{1}{tg^2x-1} \cdot \frac{1}{\cos^2 x} dx = \frac{1}{2} \left(x + \frac{1}{2} \ln \left \frac{tgx-1}{tgx+1} \right \right) \Bigg _0^{\frac{\pi}{6}} =$ $= \frac{\pi}{12} + \frac{1}{4} \ln \left \frac{tg \frac{\pi}{6} - 1}{tg \frac{\pi}{6} + 1} \right - \frac{1}{2} \cdot 0 - \frac{1}{4} \ln 1 = \frac{\pi}{12} + \frac{1}{4} \ln \frac{\sqrt{3}-3}{\sqrt{3}+3}.$	1p 1p 2p 1p