

BAREM DE EVALUARE ȘI DE NOTARE
Varianta 72

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♦ Pentru orice soluție corectă, chiar dacă este diferită de cea din barem, se acordă punctajul corespunzător.

♦ Nu se acordă fracțiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parțiale, în limitele punctajului indicat în barem.

♦ Se acordă 10 puncte din oficiu. Nota finală se calculează prin împărțirea punctajului obținut la 10.

SUBIECTUL I (30 de puncte)

1.	$C_{x-3}^0 = 1$ $C_{x-1}^2 = \frac{(x-1)!}{(x-3)! \cdot 2!} = \frac{(x-3)! \cdot (x-2)(x-1)}{(x-3)! \cdot 2} = \frac{(x-2)(x-1)}{2}$ $C_x^{x-2} = \frac{x!}{(x-x+2)! \cdot (x-2)!} = \frac{(x-2)!(x-1)x}{2(x-2)!} = \frac{(x-1)x}{2}$ Numerele $2C_{x-3}^0; C_{x-1}^2; C_x^{x-2}$ sunt în progresie aritmetică $\Rightarrow 2C_{x-3}^0 + C_x^{x-2} = 2C_{x-1}^2$ $\Rightarrow 2 + \frac{x(x-1)}{2} = 2 \cdot \frac{(x-2)(x-1)}{2} \Leftrightarrow -x^2 + 5x = 0 \Rightarrow \begin{cases} x_1 = 0 \text{ (nu convine)} \\ x_2 = 5 \end{cases} \Rightarrow S = \{5\}$	1p 1p 1p 1p 1p
2.	$x_1 + x_2 = -2(m-1)$ $x_1 x_2 = 8(m^2 - 1)$ Calculăm $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 4(m-1)^2 - 16(m^2 - 1) = 4(-3m^2 - 2m + 5)$ Fiind o funcție de gradul II în m , aceasta ia valoarea maximă egală cu $-\frac{\Delta}{4a} = \frac{16}{3}$ obținută pentru $m = -\frac{b}{2a} = \frac{2}{-6} = -\frac{1}{3}$	1p 1p 2p 1p
3.	$M = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$ Deoarece ordinal radicalului trebuie să fie un nr. natural ≥ 2 , numărul $x!$ va fi totdeauna un număr par $\Rightarrow \frac{2x^2 + x - 21}{5 - 4x - x^2} \geq 0 \Rightarrow x \in \left(-5, -\frac{7}{2}\right] \cup (1, 3]$ Cum $x \in \mathbb{N}$ și $x \geq 2 \Rightarrow A = \{2, 3\}$	1p 3p 1p
4.	$z = \frac{1+i\sqrt{3}}{1-i} = \frac{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}{\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)} \Rightarrow$ $z^{20} = \frac{2^{20}\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)}{2^{10}\left(\cos 35\pi + i\sin 35\pi\right)} = 2^{10}\left(\cos\left(\frac{10\pi}{3} - 35\pi\right) + i\sin\left(\frac{10\pi}{3} - 35\pi\right)\right) =$ $= 2^{10}\left(\cos\left(-\frac{95\pi}{3}\right) + i\sin\left(-\frac{95\pi}{3}\right)\right) = 2^{10}\left(\cos\left(-\frac{5\pi}{3}\right) + i\sin\left(-\frac{5\pi}{3}\right)\right) =$ $= 2^{10}\left(\cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}\right) = 2^{10}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2^9(1+i\sqrt{3})$	3p 2p
5.	$C_n^0 + C_n^1 + C_n^2 + \dots + C_n^n = 32$	1p

	<p>Dar $C_n^0 + C_n^1 + C_n^2 + \dots + C_n^n = 2^n$</p> <p>$\Rightarrow 2^n = 32 \Rightarrow n=5$</p> <p>$T_{k+1} = C_n^k a^{n-k} b^k \Rightarrow T_4 = C_5^3 (2x^2)^2 (-5y)^3 = -5000x^4 y^3$</p>	<p>1p</p> <p>1p</p> <p>2p</p>
6.	<p>Ecuția dreptei BC este: $x+2y-4=0$</p> <p>$A \in BC \Leftrightarrow 4a = 4 \Rightarrow a = 1$</p>	<p>3p</p> <p>2p</p>

SUBIECTUL al II-lea (30 de puncte)

1.	„ $\Rightarrow$ ” Știm	3p
a)	$\hat{a}^2 + \hat{b}^2 = \hat{0}; \hat{a}, \hat{b} \in \mathbb{Z}_3 \Rightarrow \begin{cases} pt. \hat{a} = \hat{0} \Rightarrow \hat{b}^2 = \hat{0} \Rightarrow \hat{b} = \hat{0} \\ pt. \hat{a} = \hat{1} \Rightarrow 1 + \hat{b}^2 = \hat{0} \Rightarrow \hat{b}^2 = -\hat{1} \Rightarrow \hat{b}^2 = \hat{2} \left(\text{nu are solutii} \right) \Rightarrow \\ pt. \hat{a} = \hat{2} \Rightarrow 1 + \hat{b}^2 = \hat{0} \Rightarrow \hat{b}^2 = -\hat{1} \Rightarrow \hat{b}^2 = \hat{2} \left(\text{nu are solutii} \right) \end{cases}$ $\hat{a}^2 + \hat{b}^2 = \hat{0} \Rightarrow \hat{a} = \hat{b} = \hat{0}$ <p>„$\Leftarrow$”-se verifică direct</p>	2p
b)	<p>Fie $A = \begin{pmatrix} \hat{a} & \hat{b} \\ -\hat{b} & \hat{a} \end{pmatrix} \in M$; calculăm $A^2 = \begin{pmatrix} \hat{a}^2 - \hat{b}^2 & 2\hat{a}\hat{b} \\ -2\hat{a}\hat{b} & \hat{a}^2 - \hat{b}^2 \end{pmatrix}$</p> <p>Relația $A^2 + I_2 = O_2 \Leftrightarrow \begin{cases} \hat{a}^2 - \hat{b}^2 + \hat{1} = \hat{0} (1) \\ 2\hat{a}\hat{b} = \hat{0} (2) \end{cases}$</p> <p>Din (2) $\Rightarrow \hat{a} = \hat{0}$ sau $\hat{b} = \hat{0}$</p> <p>Pentru $\hat{a} = \hat{0} \xrightarrow{in(1)} \hat{b}^2 = \hat{1} \Rightarrow \begin{cases} \hat{b} = \hat{1} \\ \hat{b} = \hat{2} \end{cases} \Rightarrow A = \begin{pmatrix} \hat{0} & \hat{1} \\ -\hat{1} & \hat{0} \end{pmatrix} \in M$ sau $A = \begin{pmatrix} \hat{0} & \hat{2} \\ -\hat{2} & \hat{0} \end{pmatrix} \in M$</p> <p>Pentru $\hat{b} = \hat{0} \xrightarrow{in(1)} \hat{a}^2 + \hat{1} = \hat{0} \Rightarrow \hat{a}^2 = -\hat{1} \Rightarrow \hat{a}^2 = \hat{2} \left(\text{nu are solutii in } \mathbb{Z}_3 \right)$</p>	2p 1p 2p
c)	<p>Mulțimea M are 9 elemente</p> <p>Fie $A = \begin{pmatrix} \hat{a} & \hat{b} \\ -\hat{b} & \hat{a} \end{pmatrix} \in M \Rightarrow \det(A) = \hat{a}^2 + \hat{b}^2 = \hat{0} \Leftrightarrow \hat{a} = \hat{b} = \hat{0} \Rightarrow A = \begin{pmatrix} \hat{0} & \hat{0} \\ \hat{0} & \hat{0} \end{pmatrix} = O_2$</p> <p>Celelalte 8 elemente din M au determinantul $\neq 0 \Rightarrow$ celelalte 8 matrice sunt inversabile</p>	2p 2p 1p
2.		
a)	$\sigma\tau = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ $\tau\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ <p>$\Rightarrow \sigma\tau \neq \tau\sigma$</p>	2p 2p 1p

b)	Not.nr.inversiunilor permutării σ cu $m(\sigma)$ și signatura cu $\varepsilon^{m(\sigma)} = (-1)^3 = -1$ $\Rightarrow m(\sigma) = 2 + 1 = 3 \Rightarrow \varepsilon^{m(\sigma)} = (-1)^3 = -1 \Rightarrow \sigma$ permutare impară $m(\tau) = 1 \Rightarrow \varepsilon^{m(\tau)} = (-1)^1 = -1 \Rightarrow \tau$ permutare impară	1p 2p 2p
c)	$\sigma\tau = \tau \Rightarrow \tau = \sigma^{-1}\tau$ Cum $\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = \sigma \Rightarrow \tau = \sigma\tau = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$	2p 2p 1p

SUBIECTUL al III-lea (30 de puncte)

1. a)	$D_f = ?$ Condiția: $1 + \frac{2}{x} > 0 \Leftrightarrow \frac{x+2}{x} > 0 \Rightarrow x \in (-\infty; -2) \cup (0; +\infty)$ Asimptota orizontală: $\lim_{x \rightarrow \pm\infty} f(x) = \ln 1 = 0 \Rightarrow ec.asimp.oriz.spre \pm \infty$ este $y=0$ Asimptote verticale: $\lim_{\substack{x \rightarrow -2 \\ x < -2}} f(x) = -\infty \Rightarrow ec.asimp.vertic.este \quad x = -2$ $\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x) = +\infty \Rightarrow ec.asimp.vertic.este \quad x = 0$	2p 1p 1p 1p															
b)	Calculăm $f'(x) = -\frac{2}{x(x+2)}$; $D_{f'} = (-\infty; -2) \cup (0; +\infty)$ Semnul lui f' : <table><tr><td>x</td><td>$-\infty$</td><td>-2</td><td>0</td><td>$+\infty$</td></tr><tr><td>$f'(x)$</td><td>-----</td><td></td><td>-----</td><td></td></tr><tr><td>$f(x)$</td><td>$0 \searrow \searrow \searrow \searrow -\infty$</td><td></td><td>$+\infty \searrow \searrow \searrow \searrow 0$</td><td></td></tr></table> $\Rightarrow$ funcția f este strict descrescătoare pe domeniul său	x	$-\infty$	-2	0	$+\infty$	$f'(x)$	-----		-----		$f(x)$	$0 \searrow \searrow \searrow \searrow -\infty$		$+\infty \searrow \searrow \searrow \searrow 0$		2p 2p 1p
x	$-\infty$	-2	0	$+\infty$													
$f'(x)$	-----		-----														
$f(x)$	$0 \searrow \searrow \searrow \searrow -\infty$		$+\infty \searrow \searrow \searrow \searrow 0$														
c)	$x_n = \frac{f(1) + f(2) + f(3) + + f(n)}{n} =$ $\frac{\ln\left(1 + \frac{2}{1}\right) + \ln\left(1 + \frac{2}{2}\right) + \ln\left(1 + \frac{2}{3}\right) + + \ln\left(1 + \frac{2}{n}\right)}{n} =$ $\frac{\ln\left(\left(1 + \frac{2}{1}\right)\left(1 + \frac{2}{2}\right)\left(1 + \frac{2}{3}\right).....\left(1 + \frac{2}{n}\right)\right)}{n} =$ $\frac{\ln\left(\frac{3}{1} \cdot \frac{4}{2} \cdot \frac{5}{3} \cdot \cdot \frac{n}{n-2} \cdot \frac{n+1}{n-1} \cdot \frac{n+2}{n}\right)}{n} =$ $\frac{\ln\frac{(n+1)(n+2)}{2}}{n} = \frac{1}{n} \ln \frac{(n+1)(n+2)}{2}$	3p															

	$\Rightarrow \lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} \frac{\ln \frac{(n+1)(n+2)}{2}}{n} = 0$	2p
2. a)	$I_0 = \int_0^{\frac{\pi}{2}} (\cos x)^0 dx = \int_0^{\frac{\pi}{2}} 1 dx = x \Big _0^{\frac{\pi}{2}} = \frac{\pi}{2}$ $I_1 = \int_0^{\frac{\pi}{2}} (\cos x)^1 dx = \int_0^{\frac{\pi}{2}} \cos x dx = \sin x \Big _0^{\frac{\pi}{2}} = \sin \frac{\pi}{2} - \sin 0 = 1$	2p 3p
b)	$I_n = \int_0^{\frac{\pi}{2}} (\cos x)^n dx = \int_0^{\frac{\pi}{2}} \cos x \cdot (\cos x)^{n-1} dx = \int_0^{\frac{\pi}{2}} (\sin x)' (\cos x)^{n-1} dx =$ $= \sin x \cdot (\cos x)^{n-1} \Big _0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (n-1)(\cos x)^{n-2} \cdot (-\sin^2 x) dx =$ $= (n-1) \int_0^{\frac{\pi}{2}} (\cos x)^{n-2} \cdot (1 - \cos^2 x) dx =$ $= (n-1) \int_0^{\frac{\pi}{2}} (\cos x)^{n-2} dx - (n-1) \int_0^{\frac{\pi}{2}} (\cos x)^n dx = (n-1) I_{n-2} - (n-1) I_n$ $\Rightarrow I_n = (n-1) I_{n-2} - (n-1) I_n \Rightarrow I_n + (n-1) I_n = (n-1) I_{n-2}$ $\Rightarrow n I_n = (n-1) I_{n-2} \mid : n \neq 0$ $\Rightarrow I_n = \frac{n-1}{n} I_{n-2}, (\forall) n \in \mathbb{N}, n \geq 2$	3p 2p
c)	$f : \left[0; \frac{\pi}{2}\right] \rightarrow [0;1], f(x) = \cos x \Rightarrow$ $V = \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx = \pi \int_0^{\frac{\pi}{2}} \frac{\cos 2x + 1}{2} dx = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (\cos 2x + 1) dx = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \cos 2x dx + \frac{\pi}{2} \int_0^{\frac{\pi}{2}} dx =$ $= \frac{\pi}{4} \int_0^{\frac{\pi}{2}} 2 \cos 2x dx + \frac{\pi}{2} x \Big _0^{\frac{\pi}{2}} = \frac{\pi}{4} (\sin 2x) \Big _0^{\frac{\pi}{2}} + \frac{\pi}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4} (\sin \pi - \sin 0) + \frac{\pi}{4} = \frac{\pi}{4}$	3p 2p